Mr. Michael T. Davis Calculus	Sections 5.1 Quiz March 23, 2017

Name: Solutions

1. Determine the extreme values of the function $f(x) = 2x^2 + 8x + 7$ Domain: $(-\infty, \infty)$

$f'(x) = 4x + 8 \qquad 0 = 4x + 8 \qquad \underset{\text{S.P.}}{x = -2} \qquad f'(-2) = 0$

$y = -1$

$- - - - \quad \bullet \quad + + + +$ f' $\qquad f(-2) = -1$

-2

$f(x)$ has an abs. min of $y = -1$ at $x = -2$ since f' changes from neg. to pos.

2. Determine the extreme values of the function $f(x) = x^3 + 4x^2 - 3x + 1$

Domain: $(-\infty, \infty) \qquad f'(x) = 3x^2 + 8x - 3 \qquad f'(x) = 0$

$0 = 3x^2 + 8x - 3 \qquad 0 = (3x - 1)(x + 3)$

$\underset{\text{S.P.}}{x = \frac{1}{3}}$ & $\underset{\text{S.P.}}{x = -3} \qquad f'\left(\frac{1}{3}\right) = 0 \qquad f'(-3) = 0$

$f\left(\frac{1}{3}\right) = \frac{1}{27} + \frac{4}{9} - 1 + 1 = \frac{13}{27} \qquad f(-3) = -27 + 36 + 9 + 1 = 19$

$y = \frac{13}{27} \qquad y = 19$

$+ + + + \quad \bullet \quad - - - - \quad \bullet \quad + + +$ f'

$-3 \qquad \frac{1}{3}$

$f(x)$ has a rel. max. of $y = \frac{13}{27}$ at $x = -3$ since $f'(x)$ changes from Pos. To Neg.

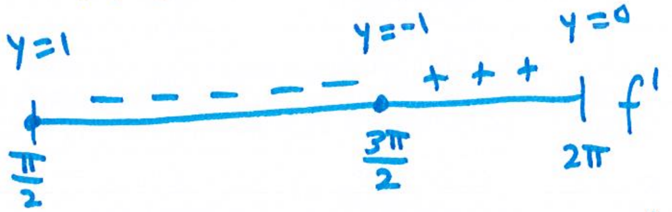
$f(x)$ has a rel. min. of $y = 19$ at $x = \frac{1}{3}$ since $f'(x)$ changes from Neg. to Pos.

3. Determine the extreme values of the function $f(x)=\sin(x)$ on $\left[\frac{\pi}{2},2\pi\right]$

$f\left(\frac{\pi}{2}\right)=1 \quad f(2\pi)=0 \quad f'(x)=\cos x$

$f'(x)=0 \quad 0=\cos x \quad \underset{\text{S.P.}}{x=\frac{\pi}{2}}$ and $\underset{\text{S.P.}}{x=\frac{3\pi}{2}}$

$f'\left(\frac{\pi}{2}\right)=0 \quad f'\left(\frac{3\pi}{2}\right)=0 \quad f\left(\frac{3\pi}{2}\right)=-1$



$f(x)$ has an abs. max. of $y=1$ at $x=\frac{\pi}{2}$ since $x=\frac{\pi}{2}$ is an endpoint and $f'(x)<0$ on the interval $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$

$f(x)$ has an abs. min. of $y=-1$ at $x=\frac{3\pi}{2}$ since f' changes from Neg. to Pos.

$f(x)$ has an rel. max of $y=0$ at $x=2\pi$ since $x=2\pi$ is an endpoint and $f'(x)>0$ on the interval $\left(\frac{3\pi}{2},2\pi\right)$.

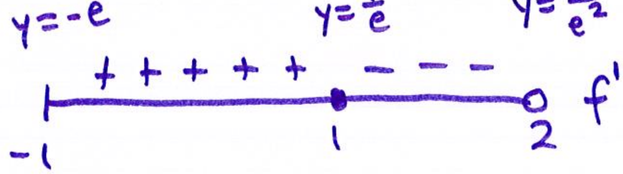
4. Determine the extreme values of the function $f(x)=\frac{x}{e^x}$ on $[-1,2)$

$f(-1)=\frac{-1}{e^{-1}}=-e \qquad f'(x)=\frac{e^x\cdot 1-xe^x}{(e^x)^2}$

$f'(x)=\frac{e^x(1-x)}{e^{2x}}=\frac{1-x}{e^x}$

$f'(x)=0 \quad 0=\frac{1-x}{e^x} \quad x=1 \quad f'(1)=0$

$f(1)=\frac{1}{e}$



$f(x)$ has an abs. min. of $y=-e$ at $x=-1$ since $x=-1$ is an endpoint and $f'>0$ on $(-1,1)$.

$f(x)$ has an abs. max. of $y=\frac{1}{e}$ at $x=1$ since f' changes from Pos. To Neg.

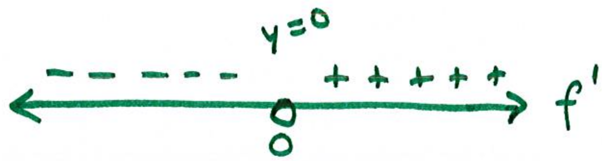
5. Determine the extreme values of the function $f(x)=x^{\frac{2}{3}}$ Domain : $(-\infty,\infty)$

$f'(x)=\frac{2}{3}x^{-\frac{1}{3}}=\frac{2}{3\sqrt[3]{x}}$

$f'(x)\neq 0$ $f'(x)=$ undefined at $x=0$

$f'(0)=$ undefined

$f(0)=0$



$f(x)$ has an abs. min. of $y=0$ at $x=0$ since f' changes from Neg. to Pos.

6. Determine the extreme values of the function $f(x)=\ln x+\frac{2}{x^2}$ on $1\le x\le 4$

$f(1)=\ln 1+\frac{2}{1^2}=0+2=2$ $f(4)=\ln 4+\frac{2}{16}\approx 1.511$

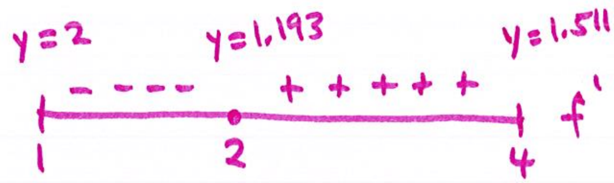
$f'(x)=\frac{1}{x}-2\cdot 2x^{-3}=\frac{1}{x}-\frac{4}{x^3}=\frac{x^2-4}{x^3}$

$f'(x)=$ undefined at $x=0$ but $x=0$ is not in $1\le x\le 4$

$f'(x)=0$ $0=\frac{x^2-4}{x^3}$ $x=2$ & $x=-2$ but $x=-2$ is not in $1\le x\le 4$

$x=2$ s.p. $f'(2)=0$

$f(2)=\ln 2+\frac{2}{2^2}\approx 1.193$



$f(x)$ has an abs. max. of $y=2$ at $x=1$ since $x=1$ is an endpoint and $f'(x)<0$ on $[1,2)$

$f(x)$ has an abs. min. of $y=1.193$ at $x=2$ since $f'(x)$ changes from Neg. to Pos.

$f(x)$ has a local max. of $y=1.511$ at $x=4$ since $x=4$ is an endpoint and $f'>0$ on $(2,4)$.